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Current Form Factors of Nucleon and Pion in Covariant Parton Model

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Abstract

We investigate in a fully covariant manner the electromagnetic(EM) and axial form factors(FFs) of the nucleon and EMFF of the pion, assuming that the current interaction with the hadron is mediated through a point-like coupling with (anti-)quarks which constitute the hadron. First we express FFs in terms of invariant amplitudes which describe the hadron off-shell quark scattering. Next we relate the asymptotic behaviour of FFs to the behaviour of the above mentioned amplitudes in the Regge limit, using the fundamental assumption of the parton model. Based on the obtained relations, we discuss the Regge trajectory functions, especially, their zero intercepts. Furthermore we make speculations on, for example, possible relations between FFs and their dependence on Q^2.

§ 1. Introduction

It has become clear that the parton models properly express the processes in which the space-like photon takes part.[1] Upon these results we investigate, in this paper, electromagnetic(EM) and axial form factors(FFs) of the nucleon and EMFF of the pion, especially, their asymptotic behaviour and possible relations between them. Until now, there are several papers[2),3),4)] in which FFs of the nucleon have been treated in the framework of the parton models. However, in these papers, only F_1(one of the EMFFs of the nucleon) has been treated and/or FFs have been calculated either in a non-covariant way or neglecting the spin of quarks*) or essencially on the on-shell approximation for quarks. In this paper we calculate the FFs in the covariant manner, without using the assumption of the validity of the on-shell approximation for quarks and relate them to the (anti-)quark nucleon (or pion) scattering amplitudes.

In the next section we express the EMFFs of the nucleon in terms of the invariant amplitudes which describe the off-shell quark scattering off the nucleon. In § 3, we investigate the asymptotic behaviour of FFs, using the fundamental assumption[5)] of the parton model that the invariant amplitudes (which are introduced in § 2) tend rapidly to zero, as the moduli of the invariant mass squares of the quarks increase. We relate, assuming the Regge behaviour for the quark nucleon scattering amplitudes, the asymptotic damping power of FFs to the Regge trajectory. In § 4, we discuss the results obtained in § 3, and make further speculations, keeping in mind the correspondence between our results and those of the

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*) A quark parton will be expressed simply as a quark.

parton model of Feynman.[1)] In the Appendix A, we express the axial FFs of the nucleon in the invariant amplitudes. In the Appendix B, we describe the EMFF of the pion in the off-shell quark pion scattering amplitudes. In the Appendix C, we give the "on-shell" quark nucleon scattering amplitudes in terms of the invariant amplitudes introduced in § 2.

§ 2. Electro-magnetic form factors of nucleon

In this section we express the EMFFs of the nucleon in terms of the invariant amplitudes describing the off-shell quark nucleon scattering. We give the formulation on the axial FFs of the nucleon and the EMFF of the pion in the Appendices A and B, respectively.

We assume that the EM current/density, $j_\mu(x)$, is expressed as *)

$$j_\mu(x) = \sum_i e_i \frac{1}{2}\left[\bar{\psi}_i(x),\ \gamma_\mu \psi_i(x)\right] \quad , \tag{1}$$

where the suffix i specifies the kind of quarks, e_i the charge of the quark i in the unit of the proton charge and $\psi_i(x)$ the field of quark i. We sandwich the Eq. (1) by the one nucleon states ~~to obtain~~ and write

$$A^\mu \equiv \langle \vec{p}',\ s' |\ j^\mu(0)\ | \vec{p},\ s\rangle \quad \delta^4(p'-p-q)$$

$$= \frac{i}{(2\pi)^3}\sqrt{\frac{M}{E(\vec{p}')}}\sqrt{\frac{M}{E(\vec{p})}}\ \bar{u}^{s'}(\vec{p}')\left[F_1(Q^2)\gamma^\mu + i\sigma^{\mu\nu}q_\nu F_2(Q^2)\right]u^s(\vec{p}) \tag{2}$$

$$\times \delta^4(p'-p-q)$$

where

$$Q^2 \equiv -q^2, \qquad E(\vec{p}) = \sqrt{\vec{p}^2+M^2}$$

and M is the nucleon mass. After some algebra, Eq. (2) can be re-expressed as follows:

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*) We use the following definitions : the metric tensor~~s~~ $g_{\mu\nu}=(+,-,-,-)$, $\gamma_5 \equiv \gamma^0\gamma^1\gamma^2\gamma^3$ and $\varepsilon^{0123}=1$.

$$A^{\mu} = -\sum_{i} e_i \int d^4k \frac{1}{(2\pi)^8} \int d^4x \, d^4x' \, \exp\{i(k\cdot x - k'\cdot x')\} \times$$
$$\times \langle \vec{p}', s' | \, T\{\psi_{i\beta}(x), \bar{\psi}_{i\alpha}(x')\} | \vec{p}, s \rangle (\gamma^{\mu})_{\alpha\beta} \quad , \tag{3}$$

where k'=k+q and p'=p+q (see Fig. 1).

Next, taking into account the parity(P) invariance, we decompose the nucleon off-shell quark scattering amplitude corresponding to Fig. 2 into invariant amplitudes as follows (in the following discussion, we neglect the suffix i which specifies the kind of quarks for simplicity) :

$$G_{\alpha\beta}(p', s'; k, k'; p, s)$$
$$= \frac{1}{(2\pi)^8} \int d^4x \, d^4x' \, \exp\{i(k\cdot x - k'\cdot x')\} \langle \vec{p}', s' | \, T\{\psi_{\alpha}(x), \bar{\psi}_{\beta}(x')\} | \vec{p}, s \rangle$$
$$= \frac{1}{(2\pi)^3} \sqrt{\frac{M}{E(\vec{p}')}} \sqrt{\frac{M}{E(\vec{p})}} \; \bar{u}^{s'}(\vec{p}') \, \{S_F'(k)\}_{\alpha\alpha'} \Big[\!\Big[A^{(1)}(s, t=-Q^2, k^2, k'^2)$$
$$\times 1 \otimes 1 + A^{(2)(\delta)} 1 \otimes g^{(\delta)}\cdot\gamma + i A^{(3)(i,j)} 1 \otimes \frac{i}{4}[g^{(i)}\cdot\gamma, g^{(j)}\cdot\gamma]$$
$$+ A^{(4)} \gamma_5 \otimes \gamma_5 + A^{(5)(\delta)} \gamma_5 \otimes \gamma_5 g^{(\delta)}\cdot\gamma + i A^{(6)(i)} \gamma_5 \otimes$$
$$\frac{i}{2}[g^{(i)}\cdot\gamma, \hat{g}^{(3)}\cdot\gamma] + B^{(1)} \Delta\cdot\gamma \otimes 1 + B^{(2)(\delta)} \Delta\cdot\gamma \otimes g^{(\delta)}\cdot\gamma$$
$$+ i B^{(3)(i,j)} \Delta\cdot\gamma \otimes \frac{i}{4}[g^{(i)}\cdot\gamma, g^{(j)}\cdot\gamma] + B^{(4)} \gamma_5 \Delta\cdot\gamma \otimes \gamma_5$$
$$+ B^{(5)(\delta)} \gamma_5 \Delta\cdot\gamma \otimes \gamma_5 g^{(\delta)}\cdot\gamma + i B^{(6)(i)} \gamma_5 \Delta\cdot\gamma \otimes$$
$$\frac{i}{2}[g^{(i)}\cdot\gamma, \hat{g}^{(3)}\cdot\gamma] \Big]\!\Big]_{\alpha'\beta'} \{S_F'(k')\}_{\beta'\beta} \; u^{s}(\vec{p})$$
$$\times \delta^4 (p + k' - p' - k) \tag{4}$$

where the suffices α' and β' accompanying the crotchet represent the $(\alpha'\beta')$ element of the latter part of the direct products of the γ matrices. On the other hand, the former part of them concerns the spinors of the nucleon. In Eq. (4), repeated indices are summed. Suffices δ and i (and j) run from zero to three and from zero to two, respectively, ~~In Eq. (4),~~ and

$$q^{(0)}=P/\tilde{Q},\quad q^{(1)}=-q/Q,\quad q^{(2)}=(\Delta-\frac{\nu}{\tilde{Q}}q^{(0)}+\frac{\beta_-}{Q}q^{(1)})/\varepsilon,$$

$$\hat{q}^{(3)}_{\mu}=\varepsilon_{\mu\nu\rho\sigma}\,q^{(0)\nu}q^{(1)\rho}q^{(2)\sigma},\qquad q^{(3)}=\gamma_5\,\hat{q}^{(3)},\qquad P=p+p',$$

$$\Delta=k+k',\qquad s=(p+k')^2,\qquad u=(p-k\;)^2,\qquad \nu=s-u$$

and

$$\beta_{\pm}=k^2\pm k'^2. \tag{5}$$

In Eq. (5), ε is chosen to satisfy the conditions (we choose $\varepsilon>0$.)

$$q^{(i)}\cdot q^{(j)}=g^{ij},\qquad \hat{q}^{(3)}\cdot q^{(i)}=0$$

and

$$\hat{q}^{(3)2}=-1. \tag{6}$$

Thus, ε is expressed as

$$\varepsilon^2=\frac{\nu^2}{\tilde{Q}^2}-2\beta_+-Q^2-\frac{\beta_-^2}{Q^2}\quad .$$

Also, the propagator for a quark $S'_F(k)$ satisfies the following spectral representation :

$$S'_F(k)=\int d\sigma\,\frac{\xi(\sigma)\,k\cdot\gamma+\zeta(\sigma)}{k^2-\sigma^2}$$

with (7)

$$\int\xi(\sigma)\,d\sigma=1\quad .$$

We find from the charge conjugation (C) invariance (therefore from the time reversal (T) invariance) that the invariant amplitudes introduced in Eq. (4) are either even or odd under the replacement of k^2 and k'^2. We enumerate below the amplitudes which are odd under this replacement :

$$A^{(2)(1)},\qquad B^{(2)(1)},\qquad A^{(3)(2,0)},\qquad B^{(3)(2,0)},\qquad B^{(4)},$$

$$A^{(5)(0)}, \quad A^{(5)(2)}, \quad A^{(5)(3)}, \quad B^{(5)(1)}, \quad A^{(6)(1)}, \quad B^{(6)(0)}$$

and

$$B^{(6)(2)}. \tag{8}$$

It is straightforward task to represent the EMFFs of the nucleon in terms of the invariant amplitudes by using Eqs. (2) $\sim$ (8). (As mentioned previously, we write the expression for a single quark with the charge 1.) The results are as follows:*),†)

$$G_M(Q^2) \equiv F_1(Q^2) + 2MF_2(Q^2)$$
$$= -\frac{\pi}{4Q\widetilde{Q}}\int d\nu\, dk^2\, dk'^2 \frac{d\sigma}{k^2-\sigma^2}\frac{d\sigma'}{k'^2-\sigma'^2} A_{G_M}(\nu, Q^2, k^2, k'^2, \sigma, \sigma') \tag{9}$$

with

$$A_{G_M}(\nu, Q^2, k^2, k'^2, \sigma, \sigma') = -\varepsilon_+ \varepsilon^2 B^{(1)} - \xi\xi' \frac{\nu}{\widetilde{Q}} \varepsilon^2 B^{(2)(0)}$$
$$- \xi\xi' \frac{\beta_-}{Q}\varepsilon^2 B^{(2)(1)} + \left\{\xi\xi'\left(\frac{\nu^2}{\widetilde{Q}^2} - \beta_+ - \frac{\beta_-^2}{Q^2}\right) - 2\zeta\zeta'\right\}\varepsilon B^{(2)(2)}$$
$$- \xi\xi'\frac{\nu Q}{\widetilde{Q}}\varepsilon B^{(2)(3)} - Q\left(\varepsilon_+ - \frac{\beta_-}{Q^2}\varepsilon_-\right)\varepsilon B^{(3)(1,2)}$$
$$- \varepsilon_- \frac{\nu}{\widetilde{Q}}\varepsilon B^{(3)(2,0)} + \xi\xi'\widetilde{Q}\varepsilon^2 B^{(5)(0)} - \xi\xi'\nu\varepsilon B^{(5)(2)}$$
$$+ \left\{\xi\xi'(Q^2+\beta_+) - 2\zeta\zeta'\right\}\frac{\widetilde{Q}}{Q}\varepsilon B^{(5)(3)} - \varepsilon_-\frac{\nu}{Q}\varepsilon B^{(6)(0)}$$
$$+ \left(\varepsilon_+ - \frac{\beta_-}{Q^2}\varepsilon_-\right)\widetilde{Q}\varepsilon B^{(6)(1)} + \varepsilon_-\frac{\widetilde{Q}}{Q}\varepsilon^2 B^{(6)(2)} \tag{10}$$

and

$$G_E(Q^2) \equiv F_1(Q^2) - \frac{Q^2}{2M}F_2(Q^2)$$
$$= -\frac{\pi\widetilde{Q}}{4MQ}\int d\nu\, dk^2\, dk'^2 \frac{d\sigma}{k^2-\sigma^2}\frac{d\sigma'}{k'^2-\sigma'^2} A_{G_E}(\nu, Q^2, k^2, k'^2, \sigma, \sigma') \tag{11}$$

with

$$A_{G_E}(\nu, Q^2, k^2, k'^2, \sigma, \sigma')$$
$$= \varepsilon_+ \frac{\nu}{\widetilde{Q}^2}\hat{A}^{(1)} + \left\{\xi\xi'\left(\frac{\nu^2}{\widetilde{Q}^2} - Q^2 - \beta_+\right) + 2\zeta\zeta'\right\}\frac{1}{\widetilde{Q}}\hat{A}^{(2)(0)}$$

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*) See the next page (page 7).

†) See the attached page (page 6-1).

†) If we assume that the integral over ν, in Eqs. (9) and (11), can be deformed/so as to enclose the s- and u- singularities without intersecting any other singularities, at least leading term of FFs as $Q^2 \longrightarrow \infty$, shown in the next section, vanishes. We assume this is not the case. We will not discuss this point any further, leaving it to Ref. 4).

$$+ \xi\xi' \frac{\nu\beta_-}{\tilde{Q}^2 Q} \hat{A}^{(2)(1)} - \xi\xi' \frac{\nu}{\tilde{Q}^2} \varepsilon \hat{A}^{(2)(2)} + \xi\xi' \frac{Q}{\tilde{Q}} \varepsilon \hat{A}^{(2)(3)}$$

$$- \left(\varepsilon_+ - \frac{\beta_-}{Q^2} \varepsilon_- \right) \frac{Q}{\tilde{Q}} \hat{A}^{(3)(0,1)} + \varepsilon_- \frac{1}{\tilde{Q}} \varepsilon \hat{A}^{(3)(2,0)} , \qquad (12)$$

where

$$\hat{A}^{(i)} \equiv A^{(i)} + \frac{2M\nu}{\tilde{Q}^2} B^{(i)} .$$

In Eqs. (10) and (12)

$$\xi \equiv \xi(\sigma) , \quad \xi' \equiv \xi(\sigma') \quad \text{and} \quad \varepsilon_\pm \equiv \xi\zeta' \pm \xi'\zeta .$$

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*) Let χ be the momentum which is perpendicular to P and q, then $\varepsilon^2/4$ equals to $-\chi^2$.

At this stage, we can easily prove, using the property (8), that A_μ(which is defined by Eq. (2)) satisfies the conservation law of the current, i.e., $q \cdot A = 0$.**)

§ 3. Asymptotic behaviour of form factors

In this section, we investigate the asymptotic behaviour of FFs which are presented in the preceding section. Throughout this section, we use the fundamental assumption[5)] that any invariant amplitudes, defined in the preceding section, tend rapidly to zero as $|k^2|$ and/or $|k'^2|$ goes to infinity. This assumption is used usually in the parton models.

In the following, we investigate the asymptotic behaviour of G_M only. We can discuss other FFs in the same way. We are interested in the region where Q^2 is very large. We separate the range of the

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**) Since the conservation law of the current is automatically satisfied in our covariant treatment, it imposes no restrictions on the invariant amplitudes, in contrast with the argument of Hughes.[3)] integration on ν into following three regions; (I) $s \approx Q^2$, u is finite, (II) $u \approx Q^2$, s is finite and (III) both $|s|$ and $|u|$ are large. We infer, from the data on the 2-body scattering of hadrons, that there is little contribution from the region III . We assume this is the case. The contribution of the region I (II) to G_M corresponds to that of the configulation where the EM current couples to the quark (anti-quark) having the large part of the momentum of the nucleon e.g., in the Breit frame.

First we treat the contribution of the region I. We decide the asymptotic behaviour of invariant amplitudes under the limit,

$$s \approx Q^2 \longrightarrow \infty, \quad u,\ k^2 \text{ and } k'^2 \text{ are finite,} \tag{13}$$

by the following procedure. We define the scattering amplitudes of the reaction: nucleon + on-shell anti-quark(with mass m) $\longrightarrow$ nucleon + on-shell anti-quark(with mass m') as follows*) (see Fig. 3) :

$$\begin{aligned}
&\frac{1}{(2\pi)^6}\sqrt{\frac{M}{E(\vec{p}\,')}}\sqrt{\frac{M}{E(\vec{p})}}\sqrt{\frac{m}{e(\vec{k})}}\sqrt{\frac{m'}{e'(\vec{k}')}}\ \bar{u}(\vec{p}\,')\,\bar{v}(-\vec{k}) \\
&\Big[\!\Big[H^{(1)}(s,t)\, 1 \otimes 1 + H^{(2)}\, 1 \otimes P\cdot\gamma + H^{(3)}\gamma_5 \otimes \gamma_5 + H^{(4)}\gamma_5 \otimes \gamma_5 P\cdot\gamma \\
&+ I^{(1)}\Delta\cdot\gamma \otimes 1 + I^{(2)}\Delta\cdot\gamma \otimes P\cdot\gamma + I^{(3)}\gamma_5\Delta\cdot\gamma \otimes \gamma_5 + I^{(4)}\gamma_5\Delta\cdot\gamma \otimes \gamma_5 P\cdot\gamma \Big]\!\Big] \\
&\times u(\vec{p})\, v(-\vec{k}')\, \delta^4(p+k'-p'-k)
\end{aligned} \tag{14}$$

where

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*) The reason why we change the mass of the anti-quark between the initial-and final-state is as follows. Since the mass of the anti-quark in the initial- and final-state must be equal, there are several invariant amplitudes which contribute to FFs but do not appear in the expression of the on-shell amplitudes. In order to overcome this difficulty we assume that their behaviour in the region under consideration agrees with the behaviour which is decided by our

(continued to the next page 8-1)

procedure.

In the case where the quarks can not escape from hadrons, our following argument becomes rather artificial. It is highly desirable to investigate the Reggeization problem of Green's functions containing bound states.

$$e(\vec{k}) = \sqrt{\vec{k}^2 + m^2} \quad \text{and} \quad e'(\vec{k}') = \sqrt{\vec{k}'^2 + m'^2}.$$

These amlitudes can be represented by the linear combinations of the invariant amplitudes*) defined by Eq. (4). We give the results in the Appendix C.

It is known[6] by the discussion on the Reggeization that the amplitudes $H^{(i)}$'s and $I^{(i)}$'s behave as $(Q^2)^{\alpha(u)-1}$ **) in the limit $s \approx -t \equiv Q^2 \to \infty$, u finite, where $\alpha(u)$ is a Regge trajectory function, exchanged in the u-channel (which is called the di-quark channel in the following discussions). What we want to know is the behaviour of the invariant amplitudes appearing in Eq. (4) in the limit (13). At the first sight, from Eq. (C-5), $B^{(2)(0)}$, for example, seems to behave as $(Q^2)^{\alpha(u)-1/2}$. However there are two possibilities that this behaviour would not be valid. One is that several amplitudes A's and B's, appearing in the expression of H's and I's, behave with higher power of Q^2 than those inspected above, (say $B^{(2)(0)} \sim (Q^2)^{\alpha(u)+1/2}$) and cancelling each other for any finite u and $k^2(k'^2)=m^2(m'^2)$ so that H's and I's behave as $(Q^2)^{\alpha(u)-1}$. The other is that several A's and B's behave with a lower power on Q^2 (say $B^{(2)(0)} \sim (Q^2)^{\alpha(u)-3/2}$). We assume, however, in this paper that among the B's, appearing in Eq. (10), all of B's which make main contributions to G_M in $Q^2 \longrightarrow \infty$

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*) Strictly speaking, we must consider the invariant amplitudes defined by an eqation in which the quark field operators of Eq. (4) are replaced by different kind.

**) They behave[6] differently in a region, $|u| \leq O(1/Q^2)$, but our following results remain unchanged even if we take of this fact.

(see below) do not satisfy either one or both of these possibilities.*)

Thus we can decide the behaviour of A's and B's in the limit (13) from the Regge behaviour of all H's and I's. (The Regge trajectory function $\alpha(u)$ does not depend on k^2 and k'^2 due to the factorizability of the Regge pole.) The results are that in the expression of A_{G_M} (Eq. (10)) the terms other than the 1st, 3rd and 13th terms (which behave as $(Q^2)^{\alpha(u)-1}$) behave as $(Q^2)^{\alpha(u)}$. Therefore A_{G_M} behaves in the limit (13) as $\beta_{G_M}(\bar{s}, k^2, k'^2) \times (Q^2)^{\alpha(u)}$ where $\bar{s} \equiv s-Q^2$, and β_{G_M} does not vanish identically by our assumption mentioned above. Now we have learned from the above discussions that G_M behaves, up to a factor of log Q^2, as

$$G_M(Q^2) \sim (Q^2)^{\alpha-1}, \quad \text{as } Q^2 \longrightarrow \infty \quad , \tag{15}$$

where α is the maximum value of $\alpha(u)$ in the region $u<0$. (If the function $\alpha(u)$ is a mono-increasing function of u, α equals to $\alpha(0)$. We assume this is the case.)

The contribution of the region II to G_M is given by the equation similar to Eq. (15), where α stands for $\alpha(0)$ for the nucleon-quark channel (which is called the 4-quark channel in the following).

It folloes from similar discussions that G_E, G_M^5 and G_E^5 behave as (for definitions of the axial FFs, see the Appendix ~~B~~ A.),

$$G_E, \ G_M^5 \text{ and } G_E^5 \sim (Q^2)^{\alpha-1}, \quad \text{as } Q^2 \longrightarrow \infty. \tag{16}$$

Also we obtain the result (See the Appendix B.)

$$F_\pi(Q^2) \sim (Q^2)^{\alpha_m - 3/2}, \quad \text{as } Q^2 \longrightarrow \infty \quad . \tag{17}$$

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*) We calculated, as an example, the amplitudes corresponding to Fig. 4. Though it does not hold for the case of the scaler and ps couplings, our assumption holds for the cases of the vector and axial vector couplings. (Though this example may not be suitable for the situation where the nucleon is thought to be a bound state of quarks.)

In Eq. (17) α_m stands for $\alpha_m(0)$ where $\alpha_m(u)$ is the Regge trajectory function for the pion (anti)quark channel (which is called the quark channel in the following).

§4. Discussions and speculations

In the subsection 4.1, we discuss the results obtained in the previous sections. In the subsection 4.2, we conjecture the behaviour of FFs and relations between them, keeping in mind the correspondence between our results and those of the parton model of Feynman.[1)]

4.1 Discussions

The results obtained in the preceding section show that all four FFs of the nucleon behave with the same power of Q^2 as Q^2 goes to infinity, i.e., they show that the ratio of any two FFs tends to constant values as Q^2 goes to infinity.

The contribution of the region I(II) discussed in the preceding section corresponds to that of the configulation where the current couples to the quark (anti-quark) having a large part of the momentum of the parent nucleon, e.g., in the Breit frame. However, it is usually thought that the probability of an anti-quark occupying a large part of the nucleon momentum is small compared with the one of quark. Therefore we can infer that the contribution of the region II is smaller than that of the region I. This means that the zero intercept $\alpha(0)$ of the Regge trajectory exchanged in the 4-quark channel is smaller than that of the diquark channel. This is consistent with the result of the phenomenological analysis[7)], by the quark line diagrams, of the 2-body and quasi 2-body reactions of hadrons. That is, the larger the number of quarks and anti-quarks constracting the Regge trajectory, the lower the zero intercept $\alpha(0)$ of the trajectory, though the quantum numbers of the channels under consideration are exotic. If qq and $qqq\bar{q}$ states really exist and their masses are sufficiently large, slopes of the trajectories corresponding to them are not steeper than that of the ordinary Regge trajectories appearing in the hadron physics. This means that the range of the force in the former is shorter than that in the latter.

The experiment[8] on $G_M(Q^2)$ seems to be consisteht with the behaviour of $(Q^2)^{-2}$. This shows that $\alpha(0)$ of the di-quark channel is nearly equal to -1. Experiments[9] seems to show that F_π behaves as $(Q^2)^{-1}$, which means that the zero-intercept of the quark trajectory is nearly equal to 1/2. And if this is the case this trajectory is flat since the quark, if any, lies on this trajectory. That is to say, it can be thought that quarks are not Reggeized and the pion-(anti)quark scattering in the high energies is described by the exchange of elementary quarks with form factors. (Quarks are thought to be elementary on the present stage of energy and momentum transfer.)

In concluding this section, we would like to make a comment. If the trajectory of the di-quark channel (qq channel) is that of cut generated by exchange of two Regge trajectories of the quark channel, $\alpha(0)$ must be nearly equal to zero (ignoring the quark mass dependence). Since $\alpha(0)$ seems to be equal to -1, we can say that a force is acting between quarks. The assumption $\alpha_m(0) \approx 1/2$ is essential for this inspection. This point will be discussed also in the next subsection.

4.2 Several speculations on FFs of the nucleon

It is well known from the data on deep inelastic electroproduction by SLAC-MIT group[10] that the quarks having a large part of the momentum of the proton in the infinite momentum frame is mainly the up quark. Hence we can conclude that the configulation where the current couples to up(down) quarks in the proton(neutron) mainly contributes to FFs in the limit $Q^2 \longrightarrow \infty$. For the configulation where the current couples to quarks having a large part of the momentum of the nucleon, e.g., in the Breit frame, also takes effect within this limit in our case, ~~Therefore~~ we can make the following prediction (for the notations e'_p and e'_n, see the Appendix A);

$$G's(\text{proton})/G's(\text{neutron}) \longrightarrow -2,$$

and

$$G^{5\prime}s(\text{proton})/G^{5\prime}s(\text{neutron}) \longrightarrow e'_p/e'_n \quad \text{as} \quad Q^2 \longrightarrow \infty . \qquad (18)$$

The experimental data[11] is, roughly speaking, consistent with $G_M(\text{proton})/G_M(\text{neutron}) \simeq \mu_p/\mu_n \simeq -1.5$ in the presently available region, $Q^2 \lesssim 1.75$ $(\text{GeV/c})^2$. If the prediction (18) is valid, this ratio will reach to -2 in the region with larger Q^2, say $Q^2 \gtrsim Q_0^2$.

The data[11] on G_E(proton) and G_E(neutron) are available in the region, $Q^2 \lesssim 4$ $(\text{GeV/c})^2$, and the ratio of them ~~is~~ seems to be positive. Therefore if our prediction (18) is valid, probably G_E(neutron) must turn negative at a point $Q^2 \lesssim Q_0^2$ (this value of Q_0^2 may be different from the one of the previous Q_0^2) because the ratio reaches -2 in $Q^2 \gtrsim Q_0^2$. Let us, now, estimate roughly the value of Q_0^2. We assume that the structure functions of the deep inelastic electroproduction nearly satisfy[12] the Bjorken's scaling law in the region $Q^2 \gtrsim Q_e(\omega)^2$.*) Following Feynman we discuss them in the Breit frame defined by the current and the quark coupling to it (see Fig. 5). Since p equals to $Q\omega/2$, the Bjorken's scaling law approximately holds in the region $p \gtrsim p_0 = Q_e(\omega) \times \omega/2$. Although what we want to know is the value of $Q_e(\omega)^2$ at $\omega \simeq 1$, if we choose, instead, the value of $Q_e(2)^2$ as 9 $(\text{GeV/c})^2$ from the available data[12], we obtain $p_0 \simeq 3$. If the argument of Feynman's naive parton model holds for the region where the nucleon has a larger momentum than p_0 in the Breit frame, our prediction (18) also holds in the same region. The counting picture of quarks is essential ingredient for these arguments. Then we can estimate $Q_0^2 = (2p_0)^2 \approx 36$ $(\text{GeV/c})^2$. This value of Q_0^2 is of couse very rough.

*) For the notations, see Ref. 1).

Having been explained in the beginning of this subsection, it is natural to think that $\alpha(0)$ of proton-anti up quark channel is larger than that of proton-anti down quark channel. If these two trajectories are increasing functions of the argument and are pararell, uu and dd exotic states, if any, have larger masses than the ud exotic state, if any. That is, the attractive force between u and d is stronger than that between u(d) and u(d). Of course we do not know whether this originates simply in the Coulomb force.

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Appendix A

In this Appendix we present the definition on axial FFs of the nucleon and express them in terms of the invariant amplitudes defined in § 2. We assume that the axial current density can be represented as

$$j_\mu^5(x) = \sum_i e_i' \frac{1}{2}\left[\bar{\psi}_i(x),\ i\gamma_5\gamma_\mu\psi_i(x)\right] \quad , \tag{A-1}$$

where e_i' is the axial charge of the quark i.

Then we ~~obtain~~ write

$$A_\mu^5 \equiv \langle \vec{p}',\ s' |\ j_\mu^5(0)\ | \vec{p},\ s\rangle\ \delta^4(p'-p-q)$$
$$= \frac{i}{(2\pi)^3}\sqrt{\frac{M}{E(\vec{p}')}}\sqrt{\frac{M}{E(\vec{p})}}\ \bar{u}^{s'}(\vec{p}')\left[G_M^5(Q^2)\gamma_5\gamma_\mu + k_A(Q^2)\gamma_5 q_\mu\right]u^s(\vec{p})\ \delta^4(p'-p-q) \tag{A-2}$$

In the same way as in the EM current, A_μ^5 can be explessed as

$$A_\mu^5 = -\sum_i e_i' \int d^4k \frac{1}{(2\pi)^8}\int d^4x\ d^4x'\ \exp\left\{i(k\cdot x - k'\cdot x')\right\} \times$$
$$\times \langle \vec{p}',\ s' |\ T\left\{\psi_{i\beta}(x),\ \bar{\psi}_{i\alpha}(x')\right\} | \vec{p},\ s\rangle\left(i\gamma_5\gamma_\mu\right)_{\alpha\beta}. \tag{A-3}$$

Using Eqs. (A-2), (A-3) and (4), we obtain the explessions for the axial FFs of the nucleon,

$$G_M^5(Q^2) = -\frac{i\pi}{4Q\tilde{Q}}\int d\nu\, dk^2\, dk'^2\ \frac{d\sigma}{k^2-\sigma^2}\ \frac{d\sigma'}{k'^2-\sigma'^2}\ \times$$
$$\times A_{G_M^5}(\nu,\ Q^2,\ k^2,\ k'^2,\ \sigma,\ \sigma') \tag{A-4}$$

with

$$A_{G_M^5}(\nu, Q^2, k^2, k'^2, \sigma, \sigma') = -\xi\xi'\frac{Q^2}{\tilde{Q}}\varepsilon^2 B^{(2)(0)}$$
$$+\xi\xi'\frac{\nu Q^2}{\tilde{Q}^2}\varepsilon B^{(2)(2)} - \left\{\xi\xi'(Q^2+\beta_+) + 2\zeta\zeta'\right\}\frac{Q}{\tilde{Q}}\varepsilon B^{(2)(3)}$$
$$-\varepsilon_+\frac{Q}{\tilde{Q}}\varepsilon^2 B^{(3)(0,1)} - \varepsilon_+\frac{\nu Q}{\tilde{Q}^2}\varepsilon B^{(3)(1,2)}$$
$$-\left(\varepsilon_- - \frac{\beta_-}{Q^2}\varepsilon_+\right)\frac{Q^2}{\tilde{Q}}\varepsilon B^{(3)(2,0)} + \varepsilon_-\varepsilon^2 B^{(4)} +$$

$$+\xi\xi'\frac{\nu}{\tilde{Q}}\varepsilon^2 B^{(5)(0)}+\xi\xi'\frac{\beta_-}{Q}\varepsilon^2 B^{(5)(1)}$$

$$-\left\{\xi\xi'\left(\frac{\nu^2}{\tilde{Q}^2}-\beta_+-\frac{\beta_-^2}{Q^2}\right)+2\zeta\zeta'\right\}\varepsilon\, B^{(5)(2)}+\xi\xi'\frac{\nu Q}{\tilde{Q}}\varepsilon\, B^{(5)(3)}$$

$$-\left(\varepsilon_- - \frac{\beta_-}{Q^2}\varepsilon_+\right)Q\varepsilon\, B^{(6)(0)}+\varepsilon_+\frac{\nu}{\tilde{Q}}\varepsilon\, B^{(6)(1)} \qquad (A-5)$$

and

$$G_E^5(Q^2)\equiv G_M^5(Q^2)+\frac{Q^2}{2M}k_A(Q^2) = -\frac{i\pi Q}{4M\tilde{Q}}\int d\nu\, dk^2\, dk'^2$$

$$\times\frac{d\sigma}{k^2-\sigma^2}\frac{d\sigma'}{k'^2-\sigma'^2}A_{G_E^5}(\nu, Q^2, k^2, k'^2, \sigma, \sigma') \qquad (A-6$$

with

$$A_{G_E^5}(\nu, Q^2, k^2, k'^2, \sigma, \sigma') = \left(\varepsilon_+ - \frac{\beta_-}{Q^2}\varepsilon_-\right)\tilde{A}^{(4)} - \xi\xi'\frac{\nu\beta_-}{\tilde{Q}Q^2}\tilde{A}^{(5)(0)}$$

$$-\left\{\xi\xi'\left(\beta_+ + \frac{\beta_-^2}{Q^2}\right)+2\zeta\zeta'\right\}\frac{1}{Q}\tilde{A}^{(5)(1)}+\xi\xi'\frac{\beta_-}{Q^2}\varepsilon\tilde{A}^{(5)(2)}$$

$$+\varepsilon_+\frac{1}{Q}\varepsilon\tilde{A}^{(6)(0)}-\varepsilon_+\frac{\nu}{Q\tilde{Q}}\tilde{A}^{(6)(2)}, \qquad (A-7)$$

where

$$\tilde{A}^{(i)}\equiv A^{(i)}-\frac{2M\beta_-}{Q^2}B^{(i)}.$$

The FF proportional to $\gamma_5 P_\mu$ in Eq. (A-2) must vanish identically because of the fact that the axial current $j_\mu^5(x)$ (Eq. (A-1)) is even under the C-conjugation. We can show in the same way as in the proof of current conservation in the EM current case that this FF vanishes identically.

Appendix B

In this Appendix, we give a formulation of the pion EMFF, F_π. We decompose the pion off-shell quark scattering amplitude into the invariant amplitudes as,

$$\tilde{G}_{\alpha\beta}(p'; k; k'; p) = \frac{1}{(2\pi)^8}\int d^4x\, d^4x'\, \exp\{i(k\cdot x - k'\cdot x')\}\times$$

$$\times \langle \vec{p}' | T \{ \psi_\alpha(x) \ \bar{\psi}_\beta(x') \} | \vec{p} \rangle$$

$$= \{ S_F'(k) \}_{\alpha\alpha'} \Big[\!\Big[M^{(1)}(s, t = -Q^2, k^2, k'^2)$$

$$+ M^{(2)(\delta)} \xi^{(\delta)} \cdot \gamma + i M^{(3)(i,j)} \frac{i}{4} [\xi^{(i)} \cdot \gamma , \xi^{(j)} \cdot \gamma] \Big]\!\Big]_{\alpha'\beta'}$$

$$\times \{ S_F'(k') \}_{\beta'\beta} \quad \delta^4(p + k' - p' - k) \qquad (B-1)$$

The EMFF of the pion $F_\pi(Q^2)$, defined as

$$\tilde{A}_\mu \equiv \langle \vec{p}' | \ j_\mu(0) \ | \vec{p} \rangle \delta^4(p'-p-q) \equiv P_\mu F_\pi(Q^2), \qquad (B-2)$$

can be represented, in the invariant amplitudes, as follows:

$$F_\pi(Q^2) = -\frac{\pi}{2Q\tilde{Q}} \int d\nu \, dk^2 \, dk'^2 \, \frac{d\sigma}{k^2 - \sigma^2} \, \frac{d\sigma'}{k'^2 - \sigma'^2} \times$$

$$\times A_{F_\pi}(\nu, Q^2, k^2, k'^2, \sigma, \sigma') \qquad (B-3)$$

with

$$A_{F_\pi}(\nu, Q^2, k^2, k'^2, \sigma, \sigma') = \varepsilon_+ \frac{\nu}{\tilde{Q}^2} M^{(1)}$$

$$+ \left\{ \xi\xi' \left(\frac{\nu^2}{\tilde{Q}^2} - Q^2 - \beta_+ \right) + 2\zeta\zeta' \right\} \frac{1}{\tilde{Q}} M^{(2)(0)}$$

$$+ \xi\xi' \frac{\nu\beta_-}{Q\tilde{Q}^2} M^{(2)(1)} - \xi\xi' \frac{\nu}{\tilde{Q}^2} \varepsilon M^{(2)(2)} + \xi\xi' \frac{Q}{\tilde{Q}} \varepsilon M^{(2)(3)}$$

$$- \left(\varepsilon_+ - \frac{\beta_-}{Q^2} \varepsilon_- \right) \frac{Q}{\tilde{Q}} M^{(3)(0,1)} + \varepsilon_- \frac{1}{\tilde{Q}} \varepsilon M^{(3)(2,0)} \qquad (B-4)$$

In the Eq. (B-4), $\tilde{Q}^2 = Q^2 + 4\mu^2$ (μ is the pion mass. we use the same notation $\tilde{Q}$ as in the nucleon case.) We can show, in the same way as in the case of the nucleon, that the quantity $\tilde{A}_\mu$ satisfies the current conservation law.

Appendix C

In this appendix, we represent H's and I's defined in Eq. (14) in the invariant amplitudes in Eq. (4).

$$
\begin{aligned}
H^{(1)} = {} & A^{(1)} + \frac{m_-}{Q} A^{(2)(1)} + \frac{m_+}{\varepsilon}\left(1 + \frac{m_-^2}{Q^2}\right) A^{(2)(2)} - \frac{\nu m_+}{\varepsilon Q \tilde{Q}} A^{(2)(3)} \\
& - \frac{\nu}{Q\tilde{Q}} A^{(3)(0,1)} - \frac{1}{\varepsilon Q}\left(\frac{\nu^2}{\tilde{Q}^2} - Q^2 - m_-^2\right) A^{(3)(1,2)} \\
& + \frac{\nu m_+ m_-}{\varepsilon Q^2 \tilde{Q}} A^{(3)(2,0)} \Bigg|_{k^2=m^2,\ k'^2=m'^2} .
\end{aligned}
\qquad (C-1)
$$

$$
\begin{aligned}
H^{(2)} = {} & \frac{1}{\tilde{Q}} A^{(2)(0)} - \frac{\nu}{\varepsilon \tilde{Q}^2} A^{(2)(2)} + \frac{Q^2 + m_+^2}{\varepsilon Q \tilde{Q}} A^{(2)(3)} + \frac{m_+}{Q\tilde{Q}} A^{(3)(0,1)} \\
& + \frac{\nu m_+}{\varepsilon Q \tilde{Q}^2} A^{(3)(1,2)} - \frac{m_-}{\varepsilon \tilde{Q}}\left(1 + \frac{m_+^2}{Q^2}\right) A^{(3)(2,0)} \Bigg|_{k^2=m^2,\ k'^2=m'^2} .
\end{aligned}
\qquad (C-2)
$$

$$
\begin{aligned}
H^{(3)} = {} & A^{(4)} - \frac{m_+}{Q} A^{(5)(1)} - \frac{m_-}{\varepsilon}\left(1 + \frac{m_+^2}{Q^2}\right) A^{(5)(2)} + \frac{\nu m_-}{\varepsilon Q \tilde{Q}} A^{(5)(3)} \\
& - \frac{1}{\varepsilon Q}\left(Q^2 + m_+^2 - \frac{\nu^2}{\tilde{Q}^2}\right) A^{(6)(0)} + \frac{\nu m_+ m_-}{\varepsilon Q^2 \tilde{Q}} A^{(6)(1)} - \frac{\nu}{Q\tilde{Q}} A^{(6)(2)} \Bigg|_{k^2=m^2,\ k'^2=m'^2} .
\end{aligned}
\qquad (C-3)
$$

$$
\begin{aligned}
H^{(4)} = {} & \frac{1}{\tilde{Q}} A^{(5)(0)} - \frac{\nu}{\varepsilon \tilde{Q}^2} A^{(5)(2)} + \frac{Q}{\varepsilon \tilde{Q}}\left(1 + \frac{m_-^2}{Q^2}\right) A^{(5)(3)} + \frac{\nu m_-}{\varepsilon Q \tilde{Q}^2} A^{(6)(0)} \\
& + \frac{m_+}{\varepsilon \tilde{Q}}\left(1 + \frac{m_-^2}{Q^2}\right) A^{(6)(1)} - \frac{m_-}{Q\tilde{Q}} A^{(6)(2)} \Bigg|_{k^2=m^2,\ k'^2=m'^2} .
\end{aligned}
\qquad (C-4)
$$

$$
I^{(i)} = H^{(i)}\left(A's \rightarrow B's\right), \quad i = 1, 2, 3, 4 . \qquad (C-5)
$$

In the above equations,

$m_{\pm} \equiv m \pm m'$,

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Figure Captions

Fig. 1. Schematic representation of the form factor, corresponding to Eq. (3). Double lines, single lines and wavy line show the nucleons, the quarks and the photon, respectively.

Fig. 2. Nucleon off-shell quark scattering diagram.

Fig. 3. Kinematics for the process, nucleon+on-shell quark(with mass m) $\longrightarrow$ nucleon+on-shell quark(with mass m').

Fig. 4. Born diagrams for the process shown in Fig. 2. Dashed line shows the exchanged particle(scalar,ps,vector or axial vector).

Fig. 5. Nucleon current scattering diagram in the Breit frame defined by the current and the quark to which the current couples.

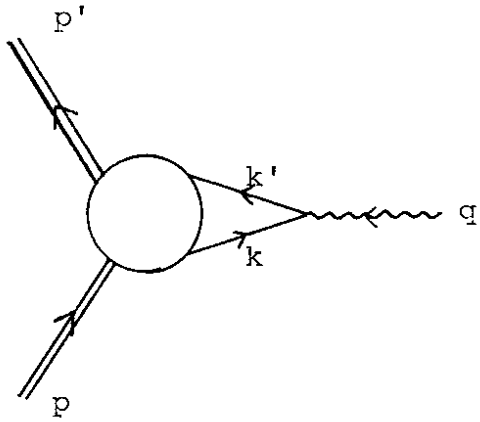

Fig.1

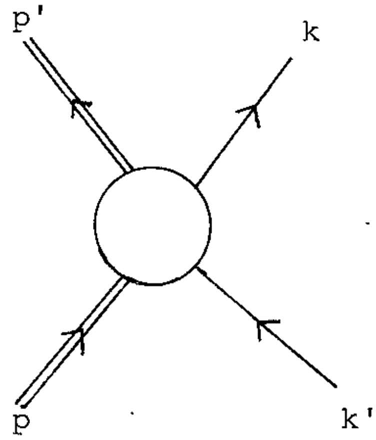

Fig.2

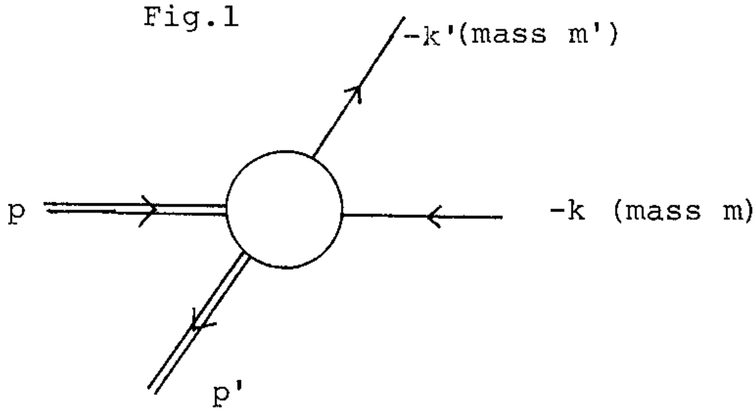

Fig.3

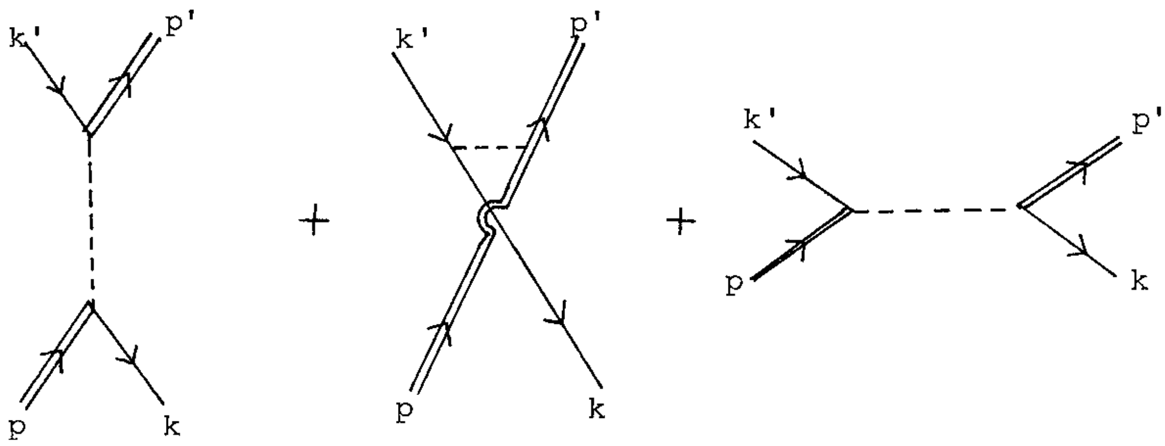

Fig.4

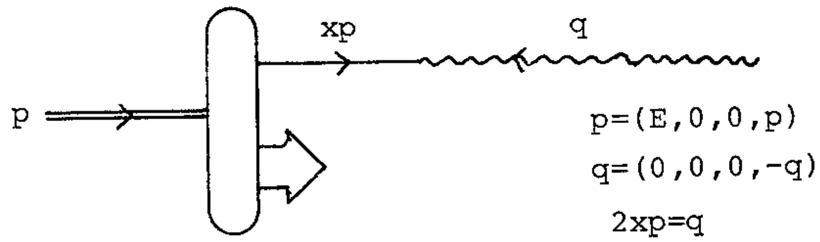

Fig.5